Bulletin of the Seismological Society of America, Vol. 79, No. 3, pp. 645-654, June 1989

ESTIMATION OF EARTHQUAKE HAZARD PARAMETERS FROM INCOMPLETE DATA FILES. PART I. UTILIZATION OF EXTREME AND COMPLETE CATALOGS WITH DIFFERENT THRESHOLD MAGNITUDES

BY A. KIJKO AND M. A. SELLEVOLL

ABSTRACT

The maximum likelihood estimation of earthquake hazard parameters (maximum regional magnitude, m_{max}, earthquake activity rate λ, and b parameter in the Gutenberg-Richter equation) is extended to the case of mixed data containing large historical events and recent complete observations. The method accepts variable quality of complete data in different parts of a catalog with different threshold magnitude values. As an illustration, the procedure is applied for the estimation of seismicity parameters in the area of Calabria and eastern Sicily.

INTRODUCTION

The available earthquake catalogs usually contain two types of information: macroseismic observations of major seismic events that occurred over a period of a few hundred years, and complete instrumental data for relatively short periods of time (the last 50 years at the most). The methods which are generally used for the estimation of seismic activity parameters (parameter b in the Gutenberg-Richter equation, earthquake activity note λ, and m_{max}) are not suitable for this type of data. Because of incompleteness of the macroseismic part of a catalog or, more exactly, because of difficulties in estimating its growing incompleteness in earlier times, the highly efficient methods of Weichert (1980) or Dong *et al.* (1984a, b) are not always applicable.

The most suitable methods for analyzing the macroseismic part of the catalog are extreme distributions, extended to allow for varying time intervals from which maximum magnitudes are selected. Assuming that this part of the catalog contains only the largest seismic events, and having the possibility of dividing the catalog into time intervals of different lengths, we can in practice analyze all the macroseismic data. Of course, we can also take into account the other, complete part of the catalog by selecting from it the largest events which occurred in relatively short time intervals (usually of 1 year duration). This method of incorporating the incomplete part of the catalog into the analysis is very far from being optimum, as a great deal of information contained in small shocks is wasted.

Another method for estimating the seismic activity parameters is to reject the macroseismic observations that are incomplete and to use any standard method for the data from the other, complete part of the catalog. It is obvious that this procedure is also highly ineffective, as the quantitative assessment of recurrence of strong seismic events based on observations over a short period of time is burdened with large errors (Knopoff and Kagan, 1977; Dong *et al.*, 1984a).

This paper presents a different approach, making it possible to combine the information contained in the macroseismic part of the catalog (strong events) with that contained in the more complete younger parts, of the catalog (Fig. 1).

The procedure is applied for the estimation of the seismicity parameters in the area of Calabria and eastern Sicily.

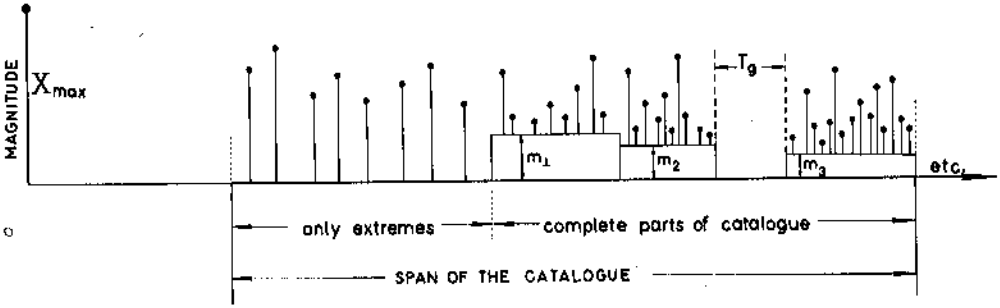

FIG. 1. An illustration of the data, which can be used to obtain basic seismic hazard parameters by the proposed procedure. Our approach permits the combination of the largest earthquakes with complete data of variable threshold magnitudes. It makes possible the application of the largest known historical earthquake X_{max} that occurred before our catalog begins; it also accepts "gaps" (as T_g; or, for example, when records are missing or seismic networks were not in operation).

Extreme Magnitude Distribution as Applied to the Macroseismic Part of the Catalog

Let us accept an assumption of the Poisson occurrence of earthquakes with the activity rate λ and the doubly truncated Gutenberg-Richter distribution $F(x)$ of earthquake magnitude x. The doubly truncated exponential distribution can be represented by the equation (Page, 1968; Cosentino *et al.*, 1977),

$$F(x) = \Pr(X \leqq x) = \frac{A_1 - A(x)}{A_1 - A_2}, \quad m_{\min} \leqq x \leqq m_{\max}, \tag{1}$$

where $A_1 = \exp(-\beta m_{\min})$, $A_2 = \exp(-\beta m_{\max})$, $A(x) = \exp(-\beta x)$, $m_{\max}$ is the maximum regional magnitude value, $m_{\min}$ is the threshold magnitude, and β is a parameter. The above assumption implies that earthquakes of magnitudes greater than x can be represented by a Poisson process with the mean rate of occurrence $\lambda[1 - F(x)]$, where λ is the activity rate corresponding to the threshold magnitude $m_{\min}$ (Benjamin and Cornell, 1970). Thus, the probability that X, the largest magnitude within a period of t years, will be less than some specified magnitude x is given by

$$G(x \mid t) = \Pr(X \leqq x) = \exp\left[-\nu_0 t\left(\frac{A_2 - A(x)}{A_2 - A_{10}}\right)\right], \tag{2}$$

where $\nu_0 = \lambda[1 - F(m_0)]$, $A_{10} = \exp(-\beta m_0)$, and m_0 is the threshold magnitude for the extreme part of catalog. ($m_0 \geqq m_{\min}$).

The resulting probability (2) is doubly truncated. From the definition of A_{10} and A_2 it follows that for $m_{\max} \to \infty$, $A_2 \to 0$, and for $m_0 = m_{\min} = 0$, $A_{10} = 1$. Thus for $A_{10} = 1$, $A_2 = 0$, and and $t = 1$, equation (2) becomes:

$$G(x) = \exp(-\lambda \exp(-\beta x)], \tag{3}$$

which is equivalent to the first Gumbel's asymptote extremes (Tinti and Mulargia, 1985a).

In the case discussed, the data for determination of seismicity parameters are the largest earthquake magnitudes $\mathbf{X}_0 = (X_{01}, \ldots, X_{0n_0})$, selected from the first part of the catalog, from time intervals $\mathbf{t} = (t_1, \ldots, t_{n_0})$. The seismicity parameters sought are $\boldsymbol{\Theta} = (\beta, \lambda)$ and $m_{\max}$. Then from equation (2) it follows that the likelihood function of $\boldsymbol{\Theta}$ is (Kijko and Dessokey, 1987)

$$L_0(\boldsymbol{\Theta} \mid \mathbf{X}_0) = \prod_{i=1}^{n_0} g(X_{0i}, t_i \mid \boldsymbol{\Theta}), \tag{4}$$

where

$$\ln g(x, t \mid \boldsymbol{\Theta}) = \frac{A_2 - A(x)}{A_{10} - A_2} + \ln \frac{\nu_0 \beta t}{A_{10} - A_2} - \beta x. \tag{5}$$

Combination of Extreme and Complete Catalogs with Different Threshold Magnitudes

Let us assume that the second part of the catalog can be divided into s subcatalogs. Each of those with a time span T_i is complete starting from the known threshold magnitude m_i, ($i = 1, \ldots, s$). Let us also assume, that the values $\mathbf{X}_i = (X_{i1}, \ldots, X_{in_i})$ denote magnitudes from the ith subcatalog, where by definition $X_{ij} \geqq m_i$, $i = 1, \ldots, s$; $j = 1, \ldots, n_i$.

If the size of seismic events is independent of their number, the likelihood function of $\boldsymbol{\Theta}$ for each subcatalog can be written as a product of two functions:

$$L_i(\boldsymbol{\Theta} \mid \mathbf{X}_i) = L_{i\beta} * L_{i\lambda}. \tag{6}$$

The likelihood function of β, $L_{i\beta}$, is well known in seismology. The assumption that the earthquake magnitude x is a random variable distributed according to the doubly truncated Gutenberg-Richter equation (1) generates $L_{i\beta}$ in the following form (Page, 1968; Cosentino *et al.*, 1977):

$$L_{i\beta} = \beta^{n_i} \exp\left(-\beta \sum_{j=1}^{n_i} X_{ij}\right) / (A_{1i} - A_2)^{n_i}, \tag{7}$$

where $A_{1i} = \exp(-\beta m_i)$, $i = 1, \ldots$, s.

Assuming in addition that the number of earthquakes per unit time is a Poisson random variable, the uncertainty in the activity rate of the ith subcatalog, ν_i, is described by the likelihood function as

$$L_{i\lambda} = \text{const} \exp(-\nu_i T_i)(\nu_i T_i)^{n_i}, \tag{8}$$

where const is normalizing factor

$$\nu_i = \lambda[1 - F(m_i)] \tag{9}$$

and λ is the activity rate corresponding to the threshold magnitude $m_{\min} = \min(m_i)$, $i = 0, \ldots, s$.

Equations (7) to (9), together with equation (6), define the likelihood function of $\boldsymbol{\Theta}$, the parameters for each subcatalog.

According to the principle of combination of data (Rao, 1973), the joint likelihood based on all data, i.e., the likelihood function for the whole span of the catalog, is given by

$$L(\Theta \mid \mathbf{X}) = \prod_{i=0}^{s} L_i(\Theta \mid \mathbf{X}_i). \tag{10}$$

Parameter Estimation

In order to estimate the parameters $\Theta = (\beta, \lambda)$, the maximum likelihood method is used. Putting $\partial \ln L(\Theta \mid \mathbf{X})/\partial\lambda = 0$ and $\partial \ln L(\Theta \mid \mathbf{X})/\partial\beta = 0$, after cumbersome calculations we obtain

$$\frac{1}{\lambda} = \phi_1{}^E + \phi_1{}^C \tag{11a}$$

$$\frac{1}{\beta} = \langle X \rangle - \phi_2{}^E - \phi_2{}^C + \lambda[\phi_3{}^E + \phi_3{}^C] \tag{11b}$$

where

$$\phi_1{}^E = r_0 B_1,$$
$$\phi_2{}^E = r_0(E(m_0, m_{\max}),$$
$$\phi_3{}^E = r_0 B_2 + \phi_2{}^E B_1,$$
$$\phi_1{}^C = \sum_{i=1}^{s} T_i C_i / n,$$
$$\phi_2{}^C = \sum_{i=1}^{s} r_i[E(m_i, m_{\max}) + D_i/C_i], \text{ and}$$
$$\phi_3{}^C = \sum_{i=1}^{s} D_i T_i / n.$$

$\langle X \rangle$ is equal to the mean earthquake magnitude calculated from the extreme and complete parts of the catalog, $n = \sum_{i=0}^{s} n_i$ is the total number of earthquakes, $r_i = n_i/n$, and

$$B_1 = (\langle t \rangle A_2 - \langle tA \rangle)/(A_2 - A_1),$$
$$B_2 = (\langle tX_0 A \rangle - \langle t \rangle m_{\max} A_2)/(A_2 - A_1),$$
$$C_i = 1 - F(m_i),$$
$$D_i = E(m_{\min}, m_i) - E(m_{\min}, m_{\max})F(m_i), \quad i = 1, \ldots, s,$$
$$E(x, y) = [xA(x) - yA(y)]/[A(x) - A_2].$$

In addition,

$$\langle t \rangle = \sum t_i / n_0,$$
$$\langle tA \rangle = \sum t_i * A(X_{0i})/n_0,$$
$$\langle tX_0 A \rangle = \sum t_i * X_{0i} * A(X_{0i})/n_0,$$

where the summation is from $i = 1, \ldots, n_0$.

The indexes E and C in equations (11) are introduced in order to distinguish different sources of functions ϕ. If they follow from the extreme part of the catalog, they are marked as E. Otherwise they follow from the complete parts of the catalog and are marked as C.

Replacing λ in equation (11b) by λ calculated from equation (11a), we obtain one equation dependent only on the parameter β, which for a given $m_{\max}$, can easily be solved by an iterative scheme. In order to understand how general the derived equations are, let us discuss some special cases.

The case $s = 1$, $r_0 = 0$ ($n_0 = 0$) and $m_{\min} = m_1$ implies that the extreme magnitudes are not taken into consideration, and the catalog is composed of only one complete part.

In this case equation (11) takes the form

$$\frac{1}{\lambda} = \frac{T}{n} \tag{12a}$$

$$\frac{1}{\beta} = \langle X \rangle - (m_{\max} A_2 - m_{\min} A_1)/(A_2 - A_1) \tag{12b}$$

Equation (12a) provides the well-known maximum likelihood estimation of the parameter of Poisson distribution. Equation (12b) takes the place of Page's (1968) formula for the maximum likelihood evaluation of β. Assuming additionally that $m_{\max} \to \infty$, equation (12b) replaces the well-known Aki (1965) and Utsu (1965) formula

$$\frac{1}{\beta} = \langle X \rangle - m_{\min}.$$

Let us discuss the second special case, when $r_0 = 1$ and $m_{\min} = m_0$. Such an assumption implies that $n_i = 0$ ($i = 1, \ldots, s$) and the complete parts of the catalog are not taken into consideration. It may be easily verified that equation (11) yields (Kijko and Dessokey, 1987):

$$\frac{1}{\lambda} = \frac{\langle t \rangle A_2 - \langle tA \rangle}{A_2 - A_1}$$
$$\frac{1}{\beta} = \langle X \rangle - \frac{\langle tX_0 A \rangle - \langle t \rangle A_2 m_{\max}}{\langle tA \rangle - \langle t \rangle A_2}. \tag{13}$$

Equation (13) can be used for the maximum likelihood estimation of β and λ in the case when input data are limited to maximum magnitudes taken from unequal time intervals. An additional assumption that $t = t_i =$ constant (magnitudes are taken from equal time intervals) reduces (13) to Kijko's (1984) formula of the first Gumbel truncated distribution. Finally, for large $m_{\max}$, equation (13) is reduced to the maximum likelihood estimation of β in the first Gumbel distribution (Kimball, 1946).

Formula (11) provides two equations for the maximum likelihood estimation of β and λ. From the formal point of view, the maximum likelihood estimate of $m_{\max}$ is simply the largest observed earthquake magnitude $X_{\max}$. This follows from the fact that $L(\mathbf{\Theta} \mid \mathbf{X})$ decreases monotonically for $m_{max} \rightarrow \infty$. Therefore, a more realistic estimation of m_{max} can be carried out by the introduction of some additional equations. According to our previous experience (Kijko, 1984; Kijko and Sellevoll, 1986; Kijko and Dessokey, 1987), the condition

$$X_{max} = \text{EXPECT}(x_{max} \mid T), \tag{14}$$

that the largest *observed* magnitude X_{max} is equal to EXPECT$(x_{max} \mid T)$, the largest *expected* magnitude in the span of the catalog T, may provide a quite satisfactory evaluation of m_{max}. The largest expected magnitude in the time interval T is given by the formula (Kijko, 1988)

$$\text{EXPECT}(x_{max} \mid T) = m_{max} - \frac{E_1(TZ_2) - E_1(TZ_1)}{\beta \exp(-TZ_2)} - m_{min}\exp(-\lambda T), \tag{15}$$

where $Z_i = -\lambda A_i/(A_2 - A_1)$, $i = 1, 2$, and $E_1(\cdot)$ denotes an exponential integral function while the span of the catalog consists of two parts, the extreme $T_0 = \sum_{i=1}^{n_0} t_i$ and complete $\sum_{i=1}^{s} T_i$.

By including condition (15) into equation (11), we obtain a set of equations determining the maximum likelihood solution, which can be solved by an iterative procedure.

Standard Errors of the Parameters

A formal estimate of the variance of $\hat{\mathbf{\Theta}} = (\hat{\beta}, \lambda$ can be obtained from the equations describing the approximate variance-covariance matrix of vector $\hat{\mathbf{\Theta}}$, $\mathbf{D}(\hat{\mathbf{\Theta}}) = \mathbf{A}^{-1}$, where in our case the matrix $\mathbf{A}$ is in the form

$$\mathbf{A} = \{a_{ij}\} = -\partial^2 \ln L/\partial\Theta_i \partial\Theta_j|_{\Theta=\hat{\Theta}}, \quad i, j = 1, 2. \tag{16}$$

Formula (16) gives a good approximation of the [redacted] variance-covariance matrix (Edwards, 1972) for sufficiently large n.

It should be noted that the formulation given here does not provide the estimation of the error of $\hat{m}_{max}$. It is cleard that the derived procedure of parameter estimation can give a abiased $\hat{m}_{max}$ when it is applied to the biased X_{max}. The problem is rather important, since the uncertainty of the maximum observed magnitude X_{max} can be as high as 0.5 of the magnitude unit when it is based on historic intensity data. Following the corresponding earlier derivations, the approximate standard deviation of $\hat{m}_{max}$ becomes (Kijko and Dessokey, 1987)

$$\hat{\sigma}_{max} = T_c(\hat{\Theta})\sigma_x, \tag{17}$$

where

$$T_c(\Theta) = \text{ABS}[\xi \exp(\xi) E_1(\xi)]^{-1},$$

$\xi = TZ_2$ and σ_x is the standard deviation of X_{max}. $T_c(\Theta)$ can be considered as a "transmission coefficient" that transmits the uncertainties in X_{max} into uncertainties of $\hat{m}_{max}$.

AN EXAMPLE

As an illustration, the described estimation procedure is used for the determination of seismicity parameters in the Calabria and eastern Sicily area limited by 36°30′ to 39°50′ N and 14°30′ to 17°20′E. All data used in this paper are taken from the excellent study of Tinti and Mulargia (1984). Our catalog contains the part with the largest earthquakes and two complete subcatalogs. The part with the largest magnitudes contains three earthquakes with magnitudes not less than 6.1 that occurred between 01/01/1631 and 04/21/1717 (Table 1). The first subcatalog, complete above magnitude 5.4, contains 7 earthquakes that occurred between 04/22/1717 and 02/05/1818. The second subcatalog, complete above magnitude 4.8, contains 38 earthquakes and covers the period from 02/06/1818 to 01/01/1979. Table 2 lists the condensed data that were used.

An application of the described parameter estimation procedure to our data gives: $\hat{\beta} = 1.93 \pm 0.31$, $\hat{\lambda} = 0.25 \pm 0.04$, $\hat{m}_{\max} = 6.80 \pm 0.35$, where $m_{\min} = 4.8$. It can be seen that $\hat{\beta} = 1.93 \pm 0.31$ corresponds to b from the Gutenberg-Richter equation equal to 0.83 ± 0.13. $\hat{m}_{\max}$ and its standard deviation were calculated according to equations (14) and (17), where the largest observed magnitude $X_{\max} = 6.6$ (Karnik, 1971) and $\sigma_x = 0.25$. The "transmission coefficient" calculated according to formula (17) is equal to 1.39.

The proposed formalism provides us with one more attractive feature. A relative quantity of information provided by each part of the catalog can be calculated. By definition (Edwards, 1972), the expected information matrix provided by experiment is of the form

$$I_{ij} = \left. \frac{\partial^2 \ln L(\mathbf{\Theta} \mid \mathbf{X})}{\partial \Theta_i \partial \Theta_j} \right|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}}. \tag{18}$$

TABLE 1

MAXIMUM MACROSEISMIC MAGNITUDES OF EARTHQUAKES FELT IN CALABRIA AND EASTERN SICILY FOR THE PERIOD FROM 01/01 1631 TO 04/21 1717*

Event	Date	Magnitude
1	03/27 1638	6.1
2	11/05 1659	6.1
3	01/11 1693	6.6

* According to Tinti and Mulargia (1984).

TABLE 2

SUMMARY OF THE INPUT DATA*

	Largest Magnitudes	Complete Parts of the Catalog	
		Subcatalog 1	Subcatalog 2
Time period	1/1/1631–4/21/1717	4/22/1717–2/05/1818	2/06/1818–1/01/1979
Number of magnitudes	3	7	38
Maximum observed magnitude	6.6		
Threshold magnitude		5.4	4.8
Average magnitude		5.74	5.24

* From Tinti and Mulargia (1984).

Thus, in our case, the rate of information of Θ_i parameter provided by the kth complete subcatalog takes the form

$$\left.\frac{\partial^2 \ln L_k(\mathbf{\Theta} \mid \mathbf{X})}{\partial \Theta_i^2}\right|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}} \Bigg/ \left.\frac{\partial^2 \ln L(\mathbf{\Theta} \mid \mathbf{X})}{\partial \Theta_i^2}\right|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}} \tag{19}$$

for $i = 1, 2$ and $k = 1, \ldots, s$.

An application of formula (19) to our data shows that the first complete subcatalog contains 24.2% and 14.6% of the total information of $\hat{\beta}$ and $\hat{\lambda}$, respectively. The second subcatalog gives 64.1% and 79.2% information respectively (Table 3). The probability that a given earthquake magnitude will not be exceeded in any year in the discussed area is shown in Figure 2.

It is interesting to compare our results with the original ones obtained by Tinti and Mulargia (1984). The best least-squares fit of the first Gumbel asymptote gives $\hat{\beta} = 1.81 \pm 0.07$, while our procedure gives 1.93 ± 0.31. The difference between these two sets of values in terms of return periods is also small, even for large magnitudes. For example the return periods for magnitude 6.0 are 49 and 51 years, respectively, as calculated by Tinti and Mulargia and by us. It is difficult to perform such a comparison for very large earthquakes. The discrepancy follows from the fact that our model takes into account the saturation of magnitude ($m_{\max}$), while the first Gumbel asymptote is unlimited. In addition, the discrepancy can be ascribed to different magnitude scales (we follow the Karnik (1972) scale) and different data sets used by Tinti and Mulargia and by us.

TABLE 3

EVALUATION OF RELATIVE AMOUNTS OF INFORMATION (EQUATION (19)) PROVIDED BY DIFFERENT PARTS OF THE CATALOG

Part of the catalog	Quantity of information (%)	
	β	λ
Extreme	11.7	6.2
Complete ≧ 5.2	24.2	14.6
Complete ≧ 4.8	64.1	79.2

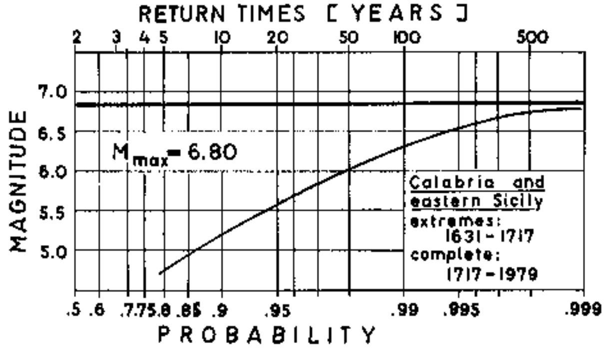

FIG. 2. Mean return periods and probability that a given magnitude will not be exceeded in any year for the discussed area.

Remarks and Conclusions

In this paper, the problem of incorporating many sources of available seismological information is presented. This is particularly useful when historical catalogs have to be combined with short periods of instrumental data.

The proposed approach is extremely flexible and provides several attractive properties. It is possible to estimate m_{max} from the largest known historical earthquakes that occurred before our catalog begins (Fig. 1). This can be achieved by substituting X_{max} by the largest known earthquake magnitude and the span of catalog by the time interval between the date of this event occurrence and the end of the catalog.

Our approach accommodates "gaps" in the extreme part as well as in the complete parts of the catalog. This follows from the fact that the procedure describing the maximum likelihood estimation does not require magnitudes taken from consecutive time intervals.

A method of seismic hazard evaluation, taking into account the different quality of available earthquake files, is presented. The proposed approach is very general and is derived from the commonly accepted assumptions and constraints related to earthquake occurrence. The described procedure permits us to calculate the maximum likelihood estimates of the mean rate λ of earthquake occurrence, the parameter b of the Gutenberg-Richter relation ($b = \beta \log e$), and the maximum regional magnitude m_{max}. With reference to the importance of λ, β, and m_{max} values for seismic hazard analysis, additional formulas are given describing the uncertainties of their estimates. The proposed procedure was applied to a set of earthquake data felt and recorded in the Calabria and eastern Sicily area, known for its high seismicity, where some of the most catastrophic earthquakes in Italy were recorded (Tinti and Mulargia 1984).

The computer programs used in our study (written in GW BASIC and FORTRAN 77 for IBM PC and compatible) can be provided if a blank 5¼″ floppy disk is sent to one of the authors.

Acknowledgments

This work was completed when one of the authors (A.K.) was working at the Institute of Seismology, University of Helsinki. He wishes to express his gratitude to all the people at the Institute for useful discussions. We are also very grateful to the anonymous reviewers for helpful and stimulating comments.

References

Aki, K. (1965). Maximum likelihood estimate of b in the formula $\log N = a - bM$ and its confidence limits, *Bull. Earthquake Res. Inst., Tokyo Univ.* **43**, 237–239.

Benjamin, J. R. and C. A. Cornell (1970). *Probability, Statistics and Decision for Civil Engineers*, McGraw-Hill, New York.

Cosentino, P., V. Ficara, and D. Luzio (1977). Truncated exponential frequency-magnitude relationship in earthquake statistics, *Bull. Seism. Soc. Am.* **67**, 1615–1623.

Dong, W. M., A. B. Bao, and H. C. Shah (1984a). Use of maximum entropy principle in earthquake recurrence relationships, *Bull. Seism. Soc. Am.* **74**, 725–737.

Dong, W. M., H. C. Shah, and A. B. Bao (1984b). Utilization of geophysical information in Bayesian seismic hazard model, *Soil Dynamics and Earthquake Engineering* **3**, 103–111.

Edwards, A. W. F. (1972). *Likelihood*, Cambridge University Press, New York, p. 235.

Karnik, V. (1971). *Seismicity of the European A/2*, D. Reidel Publishing Company/Dordrecht, Holland, p. 218.

Kijko, A. (1984). Is it necessary to construct empirical distributions of maximum earthquake magnitudes?, *Bull. Seism. Soc. Am.* **74**, 339–347.

Kijko, A. (1988). Maximum likelihood estimation of Gutenberg-Richter b parameter for uncertain magnitude values, *Pageoph* **127**, 573–579.

Kijko, A. and T. Ahjos (1985). Seismic hazard in Finland; evaluation of M_{max}, *Geophysics* **21**, 39–50.

Kijko, A. and M. Dessokey (1987). Application of the extreme magnitude distribution to incomplete earthquake files, *Bull. Seism. Soc. Am.* **77**, 1429–1436.

Kijko, A. and M. A. Sellevoll (1986). *Bayesian Estimation of Seismic Parameters for Extreme Historical and Complete Instrumental Data,* Seismo-Series No. 7, Seismological Obs. Bergen University, Norway, p. 26.

Kimball, B. F. (1946). Sufficient statistical estimation function for the parameters of the distribution of maximum values, *Ann. Math. Statists.* **17**, 299–306.

Knopoff, L. and Y. Kagan (1977). Analysis of the theory of extremal values as applied to earthquake problems, *J. Geophys. Res.* **82**, 5647–5657.

Page, R. (1968). Aftershocks and microaftershocks, *Bull. Seism. Soc. Am.* **58**, 1131–1168.

Rao, C. R. (1973). *Linear Statistical Inference and Its Application,* Ed. 2, John Wiley and Sons, New York, p. 625.

Tinti, S. and F. Mulargia (1984). Probability of occurrence of large earthquakes in Calabria and eastern Sicily on the basis of extreme value theory, *Annales Geophysicae* **2**, 529–536.

Tinti, S. and F. Mulargia (1985a). Application of extreme values approaches to the apparent magnitude distribution of the earthquakes, *Pageoph* **123**, 199–220.

Tinti, S. and F. Mulargia (1985b). Effects of magnitude uncertainties on estimating the parameters in the Gutenberg-Richter frequency-magnitude law, *Bull. Seism. Soc. Am.* **75**, 1681–1697.

Utsu, T. (1965). A method for determining the value of b in formula $\log n = a - bM$ showing the magnitude-frequency relation for earthquakes, *Geophys. Bull. Hokkaido Univ.* **13**, 99–103.

Weichert, D. H. (1980). Estimation of the earthquake recurrence parameters from unequal observation periods for different magnitudes, *Bull. Seism. Soc. Am.* **70**, 1337–1346.

INSTITUTE OF GEOPHYSICS
POLISH ACADEMY OF SCIENCES
KS. JANUSZA 64
01-452 WARSAW, POLAND
(A.K.)

SEISMOLOGICAL OBSERVATORY
UNIVERSITY OF BERGEN
5007 BERGEN, NORWAY
(M.A.S.)

Manuscript received 7 July 1986